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Transverse stability of short line-solitons in ferromagnetic media

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Abstract

In this work the propagation of nonlinear electromagnetic short waves in a ferromagnetic medium is discussed. It is shown that such waves propagate perpendicular to the magnetization density. The evolution of the wave under the influence of perturbations in one transverse dimension is considered; the asymptotic model equation governing the dynamics is a $(2 + 1)$ generalization of the well-known sine-Gordon model. We exhibit the line-soliton solution and study its transverse stability. A numerical study of the model corroborates our analytical predictions.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Wave propagation in ferromagnetic media is well known as a highly nonlinear problem. The propagation of solitons in such materials has been predicted theoretically [1–3] and observed experimentally [4–6] for a long time. Theoretical predictions, just as in other nonlinear and dispersive media, are made through two main types of model equations: long waves and modulational asymptotic models [7–10].

The latter are wave envelope soliton equations which represent nonlinear modulation of a wave train. Such nonlinear modulation is worked out by means of the slowly varying envelope approximation (SVEA), the main assumption of which is that the wave number of the wave envelope is much smaller than that of the carrier wave. The ratio of these two quantities can be used as a perturbation parameter in the analysis of the whole equations. SVEA usually leads in $(1 + 1)$ dimensions to the nonlinear Schrödinger equation (NLS) [11, 12]. The procedure provides a description of the nonlinear dynamics of the modulation.

Nevertheless SVEA possesses two serious drawbacks: the first one is that the dynamics of the wave profile (the carrier wave) itself remains absolutely unknown, and the second one is that the length of experimentally available wave packets is not always very long with regard to the carrier's wave length. So, in addition to the theoretical impossibility of understanding the wave profile dynamics, we are faced with the fact that SVEA is not always valid in practice. For example in nonlinear optics the recent development of ultrafast sources has produced pulses down to two optical cycles [13]. That has led to the development of several models able to describe wave propagation beyond the SVEA [14, 15].

The purpose of this paper is to investigate theoretically and numerically the dynamics of a nonlinear electromagnetic short wave *profile* in a ferromagnetic medium. An analogous phenomenon has been previously studied in [16] but was restricted to plane waves, i.e. to (1+1)-dimensional propagation in a bulk medium. The present paper generalizes this approach to (2+1) dimensions, which can account for volume wave propagation in thick films or plates. Indeed, if the thickness is large enough, modal dispersion can be neglected, but if it is not too large, transverse instabilities in the corresponding direction must not be taken into account. The physical relevance of a (1+1)-dimensional model to account for a (2+1)-dimensional problem has been verified experimentally in the case of magnetostatic envelope soliton propagation in yttrium iron garnet films: narrow stripes behave as a one-dimensional medium [17]. Analogously, a (2+1)-dimensional model can account for a (3+1)-dimensional problem under the above assumptions.

The present study is carried out using a generalization of a previously introduced method able to describe asymptotic short-wave dynamics [18, 19]. The paper is organized as follows. In section 2 we introduce the Landau–Lifschitz and the Maxwell equations governing the evolution of the electromagnetic wave and the magnetization density in a saturated ferrite. The analysis of the associated linear dispersion relation shows that this system can propagate short waves. To tackle the problem of nonlinear short waves, a multiple scale perturbative method is carried out in section 3 and leads to an asymptotic model equation. The model generalizes to (2+1) dimensions the universal and completely integrable (1+1) sine-Gordon equation (which has been found in [16]). This new asymptotic model constitutes the major result of the paper. The analytical study of its line-soliton solutions is then undertaken in section 4. We study their transverse stability and find a criterion which depends on the soliton itself. In section 5 the stability result is then checked numerically. Finally, section 6 is devoted to some final remarks and open issues.

2. Landau–Lifschitz and Maxwell equations: linear analysis

The system under consideration is a saturated nonconducting ferromagnetic medium where an electromagnetic wave propagates. The evolution of the magnetic field \vec{H} and the magnetization density \vec{M} is governed by two equations: the Landau–Lifschitz and the Maxwell equations. The Landau–Lifschitz equation reads, under the hypothesis of zero damping

$$\partial_t \vec{M} = -\gamma \mu_0 \vec{M} \wedge \vec{H}, \quad (1)$$

where γ is the gyromagnetic ratio and μ_0 is the magnetic permeability of the vacuum. The Maxwell equation reduces to

$$-\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) + \Delta \vec{H} = \frac{1}{c^2} \partial_t^2 (\vec{H} + \vec{M}), \quad (2)$$

where $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the speed of light with ε_0 the scalar permittivity of the medium. Damping in ferromagnetic media is in general small but not negligible. However, it can be shown that it can be neglected in the present situation; the justification as well as the analysis

taking damping into account is left for further publication. We neglect inhomogeneous exchange, since we consider bulk polaritons in a ferromagnet: in this case the wavelenghtes are large with regard to the exchange length. We also assume that the crystalline and surface anisotropy of the sample can be neglected. The quantities \vec{M}, \vec{H}, t are rescaled into $\frac{c}{\mu_0\gamma}\vec{M}, \frac{c}{\mu_0\gamma}\vec{H}$, and ct , so that the constants $\frac{\mu_0\gamma}{c}$ and c in equations (1) and (2) are replaced by 1.

To study the linear regime we look at a small perturbation of a given solution. So we linearize equations (1) and (2) about the steady state:

$$\vec{M}_0 = (m \cos \varphi, m \sin \varphi, 0), \tag{3}$$

$$\vec{H}_0 = \alpha \vec{M}_0, \tag{4}$$

where m is the normalized saturation magnetization, φ is the angle between the dominant propagation direction x and the internal magnetic field, and α represents the strength of the latter. Then we look for solutions proportional to $\exp i(kx + ly - \omega t)$:

$$\vec{M} = \vec{M}_0 + \vec{M}_1 \exp i(kx + ly - \omega t) \tag{5}$$

$$\vec{H} = \vec{H}_0 + \vec{H}_1 \exp i(kx + ly - \omega t), \tag{6}$$

where $\vec{M}_1 = (M_1^x, M_1^y, M_1^z)$ and $\vec{H}_1 = (H_1^x, H_1^y, H_1^z)$ are the real vectors, k, l are the wave numbers in the x, y directions and ω is the frequency. The dispersion relation is

$$\omega^2(\omega^2 - l^2 - k^2)^2 + m^2[(1 + \alpha)\omega^2 - \alpha(k^2 + l^2)][(l \cos \varphi - k \sin \varphi)^2 - (1 + \alpha)\omega^2 + \alpha(k^2 + l^2)] = 0. \tag{7}$$

The short-wave approximation is possible when the dispersion relation admits an expansion of the form [18, 19]

$$\omega = \frac{a}{\varepsilon} + b\varepsilon + d\varepsilon^3 + f\varepsilon^5 + \dots, \tag{8}$$

where a, b, d, f, \dots are the constants and the small parameter ε is linked to the magnitude of the wavelength through $k = k_0/\varepsilon$, which corresponds to short waves. k_0 is here some reference value of the wave number, i.e. $k_0 = \omega_r/c$, where ω_r is the ferromagnetic resonance frequency. The direction of the wave propagation is assumed to be close to the x -axis, in such a way that the y variable gives only account of a slow transverse deviation. Therefore l is assumed to be very small with respect to k , we write $l = l_0$, of order 0 with respect to ε . Substituting (8) into (7) and solving order by order, we obtain successively

- $a = k_0$
- $\varphi = \pm\pi/2$,
- $b^2 - b\left(\frac{l_0^2}{k_0} + \frac{m^2(1+\alpha)}{2k_0}\right) + \frac{l_0^2}{4k_0^2}(m^2\alpha + l_0^2) = 0$, which determines b ,
- higher order equations which determines d, f, \dots

The phase up to order ε is thus

$$\frac{1}{\varepsilon}k_0(x - t) + l_0y - \varepsilon bt, \tag{9}$$

which motivates the introduction of new variables:

$$\zeta = \frac{1}{\varepsilon}(x - Vt), \quad y = y, \quad \tau = \varepsilon t. \tag{10}$$

The variable ζ allows us to describe the shape of the wave propagating with speed V , it assumes a short wavelength about $1/\varepsilon$. The slow time variable τ accounts for the propagation

at very long time on distances very large with regard to the wavelength. The transverse variable y has an intermediate scale, as in Kadomtsev–Petviashvili (KP)-type expansions [20]. It was found that $\varphi = \pm\pi/2$, which means that the short-wave approximation is possible only if the propagation direction is perpendicular to the magnetization density. Thus the short-wave approximation is possible when the propagation direction x is perpendicular to the magnetization density. Ought to the small transverse deviation accounted for by the y variable, it means physically that the short-wave soliton will propagate in a direction close to the perpendicular to the magnetization density.

3. Nonlinear analysis: a two-dimensional short-wave equation

Through equation (10), we have put in focus the small scales dynamics in the linear limit. We now turn to the nonlinear aspect, which constitutes the main purpose of this paper. Equation (10) allows us to introduce rescaled space and time operators as

$$\frac{\partial}{\partial x} = \frac{1}{\varepsilon} \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial t} = \frac{-V}{\varepsilon} \frac{\partial}{\partial \zeta} + \varepsilon \frac{\partial}{\partial \tau}. \quad (11)$$

The fields \vec{M} and \vec{H} are expanded in power series of ε as

$$\vec{M} = \vec{M}_0 + \varepsilon \vec{M}_1 + \varepsilon^2 \vec{M}_2 + O(\varepsilon^3), \quad (12)$$

$$\vec{H} = \vec{H}_0 + \varepsilon \vec{H}_1 + \varepsilon^2 \vec{H}_2 + O(\varepsilon^3), \quad (13)$$

where $\vec{M}_0, \vec{H}_0, \vec{M}_1, \vec{H}_1, \dots$ are the functions of (ζ, y, τ) . The boundary conditions are

$$\lim_{\zeta \rightarrow -\infty} \vec{H}_j = \lim_{\zeta \rightarrow -\infty} \vec{M}_j = \vec{0}, \quad (14)$$

for all $j \geq 1$, and

$$\lim_{\zeta \rightarrow -\infty} \vec{H}_0 = \alpha \lim_{\zeta \rightarrow -\infty} \vec{M}_0 = \alpha \begin{pmatrix} 0 \\ m \\ 0 \end{pmatrix}. \quad (15)$$

Expansions (12), (13) and operators (11) are substituted into equations (1) and (2) and solved order by order. At leading order $1/\varepsilon^2$ in (2) and $1/\varepsilon$ in (1), it is found that

- \vec{M}_0 is uniform,
- $H_0^x = 0$,
- H_0^y and H_0^z remain free if $V = 1$.

We consider from now on this value of the velocity V . In physical units, the wave velocity is thus the speed of light in the medium c .

At order $1/\varepsilon$, we get

$$\vec{M}_1 = m \int_{-\infty}^{\zeta} H_0^z d\zeta' \vec{e}_x, \quad (16)$$

where \vec{e}_x denotes the unitary vector in the x direction, and

$$H_1^x = - \int_{-\infty}^{\zeta} (\partial_y H_0^y + m H_0^z) d\zeta'. \quad (17)$$

At order ε^0 ,

$$\partial_\zeta \vec{M}_2 = \begin{pmatrix} m H_1^z \\ -M_1^x H_0^z \\ -m H_1^x + M_1^x H_0^y \end{pmatrix} \quad (18)$$

is computed from equation (1) and used in order to eliminate \vec{M}_2 from equation (2), which yields the following conditions:

$$-\partial_y H_1^x + 2\partial_\tau H_0^y + M_1^x H_0^z = 0, \tag{19}$$

$$\partial_y^2 H_0^z + 2\partial_\zeta \partial_\tau H_0^z + m\partial_\zeta H_1^x - \partial_\zeta (M_1^x H_0^y) = 0. \tag{20}$$

Using (16) and (17), (19) and (20) reduce to

$$\int_{-\infty}^\zeta \partial_y^2 H_0^y d\zeta' + \partial_y M_1^x + 2\partial_\tau H_0^y + \frac{1}{m} M_1^x \partial_\zeta M_1^x = 0, \tag{21}$$

$$\frac{-1}{m} \partial_y^2 \partial_\zeta M_1^x - \frac{2}{m} \partial_\zeta^2 \partial_\tau M_1^x + m\partial_y H_0^y + m\partial_\zeta M_1^x + \partial_\zeta (M_1^x H_0^y) = 0. \tag{22}$$

Equations (21) and (22) yield the sought asymptotic model. We check that the following terms in the perturbative expansion can be computed, which ensures the validity of the asymptotic. The question of the effect of higher order terms might rise. In perturbations series associated with multi-scales methods, they usually correspond to a renormalization of the soliton speed [21], and therefore we can expect that they will not qualitatively modify the results of the present paper, at least for evolution times of the order of magnitude considered.

Setting

$$A = \frac{-H_0^y}{m} - 1, \quad B = \frac{M_1^x}{2m}, \tag{23}$$

$$X = \frac{-m}{2}\zeta, \quad Y = m\tau, \quad T = m\tau, \tag{24}$$

reduces equations (21) and (22) to

$$\partial_X \partial_T B = AB + \partial_Y^2 B - \int^X \partial_Y A, \tag{25}$$

$$\partial_X \partial_T A = -\partial_X (B \partial_X B) + \partial_Y^2 A + \partial_X \partial_Y B, \tag{26}$$

where $\int^X f$ denotes a primitive of f vanishing as $X \rightarrow +\infty$.

When the variations in the y direction are omitted ($\partial_Y = 0$), system (25), (26) coincides with the model derived in [16], which reduces to the sine-Gordon equation. This reduction cannot be generalized to the present (2+1)-dimensional situation. Some symmetry is recovered in the system by setting

$$A = \partial_X C, \tag{27}$$

which reduces system (25), (26) to

$$C_{XT} = -BB_X + C_{YY} + B_Y, \tag{28}$$

$$B_{XT} = BC_X + B_{YY} - C_Y, \tag{29}$$

where the subscripts denote partial derivatives (i.e. $C_Y = \partial_Y C$, and so on).

System (28), (29) derives from the following Lagrangian density:

$$\mathcal{L} = \frac{1}{2}C_X C_T + \frac{1}{2}B_X B_T - \frac{1}{2}(C_Y)^2 - \frac{1}{2}(B_Y)^2 + C B_Y + \frac{1}{2}C_X B^2, \tag{30}$$

through

$$\frac{\delta \mathcal{L}}{\delta C} = 0, \quad \frac{\delta \mathcal{L}}{\delta B} = 0. \tag{31}$$

4. The soliton and its transverse stability

Neglecting transverse variations, a kink solution of (28), (29) is [16]

$$B = 2w \operatorname{sech} z, \quad C = w(2 \tanh z - z), \quad z = (x - wt), \quad (32)$$

where the velocity w of the kink is an arbitrary real parameter. A more general plane solitary wave can be deduced from solution (32), as

$$B = p + 2w \operatorname{sech} z, \quad C = w(2 \tanh z - z), \quad z = x + py - (w - p^2)t, \quad (33)$$

where p is an arbitrary real parameter. For nonzero p , w does not represent the soliton velocity any more.

The stability of the line-soliton (32) with respect to slow transverse perturbations is studied following the approach of [22] for the line-solitons of KP. We introduce a slow transverse perturbation θ of the variable z and expand the fields in a perturbation series about the line-soliton as

$$B = B_0 + \eta B_1 + \dots, \quad C = C_0 + \eta C_1 + \dots, \quad (34)$$

where η is a small parameter and

$$B_0 = 2w \operatorname{sech} z, \quad C_0 = w(2 \tanh z - z), \quad z = x - wt + \theta(\eta y, \eta t). \quad (35)$$

The variables $\eta y, \eta t$ are denoted by Y and T respectively. The expansion is then substituted into system (28), (29) and solved order by order. At order η^0 , we get the equations satisfied by B_0 and C_0 . At order η^1 , we get

$$C_{1z} = (\gamma_z + 1)\theta_T - \beta\theta_Y + \beta B_1 + \kappa, \quad (36)$$

where we have set

$$\beta = \frac{B_0}{w}, \quad \gamma = \frac{C_0}{w}, \quad (37)$$

and κ is a constant with respect to z . Some attention must be paid to the integration constant. Indeed, β vanishes quickly as z tends to $\pm\infty$, but γ (or C_0) does not. Since

$$\gamma_z = \frac{1}{2}\beta^2 - 1, \quad (38)$$

we see that $\gamma_z + 1$ goes to zero at infinity. We can assume that C_{1z} vanishes also at infinity, which yields $\kappa = 0$. Substituting expression (36) of C_{1z} into the other equation obtained at this order yields the following equation for B_1 :

$$L \cdot B_1 = \left(\partial_z^2 + \frac{3}{2}\beta^2 - 1\right) B_1 = \left(\frac{3}{2}\beta^2 - 1\right)\theta_Y - (\beta^3 - \beta)\theta_T. \quad (39)$$

We made use of the relation

$$\beta_{zz} = \frac{-\beta^3}{2} + \beta, \quad (40)$$

which, as (38), follows from the equations at order 0. In order to solve equation (39), we observe that

$$L \cdot 1 = \frac{3}{2}\beta^2 - 1, \quad (41)$$

$$L \cdot \beta = \beta^3, \quad (42)$$

$$L \cdot \beta_z = 0, \quad (43)$$

$$L \cdot z\beta_z = 2\beta - \beta^3. \quad (44)$$

Thus

$$B_1 = \theta_Y + \frac{1}{2}(z\beta_z - \beta)\theta_T. \tag{45}$$

Then (36) reduces to

$$C_{1z} = \frac{1}{2}z\beta\beta_z\theta_T. \tag{46}$$

At order η^2 , a first equation yields

$$C_{2z} = \frac{1}{w}C_{1T} - (\gamma\theta_Y)_Y - \frac{1}{w} \int^z B_{1Y} + \frac{1}{2w}B_1^2 + \beta B_2 + K, \tag{47}$$

where K is a constant with respect to z . Though the adequate modification of K , the notation $\int^z B_{1Y}$ can hold for any primitive of B_{1Y} . Substituting expression (47) into the second equation obtained at this order yields

$$QB_2 = G_z, \tag{48}$$

where the operator Q is defined by

$$Q = -w\partial_z L = -w(\partial_z^3 + \frac{3}{2}\partial_z\beta^2 - \partial_z), \tag{49}$$

and

$$G = -B_{1zT} + B_{0YY} - C_{1Y} + B_1C_{1z} + \beta \left[C_{1T} - w(\gamma\theta_Y)_Y - \int^z B_{1Y} + \frac{1}{2}B_1^2 + K \right]. \tag{50}$$

The adjoint operator Q^A of Q for the scalar product defined by $(f|g) = \int_{-\infty}^{+\infty} f(z)g(z) dz$ is

$$Q^A = w(\partial_z^3 + \frac{3}{2}\beta^2\partial_z - \partial_z). \tag{51}$$

We check using (40) that $Q^A\beta = 0$, and hence

$$(\beta|G_z) = (\beta|QB_2) = (Q^A\beta|B_2) = (0|B_2) = 0. \tag{52}$$

The scalar product in (52) is computed using expansion (50). Since β is even and vanishes quickly at infinity, many terms are zero, and relation (52) reduces to

$$H\theta_{TT} + I\theta_{YY} + J\theta_{TY} = 0, \tag{53}$$

with

$$\begin{aligned} H &= \frac{1}{4} \int_{-\infty}^{+\infty} \beta^2 dz - \frac{3}{16} \int_{-\infty}^{+\infty} \beta^4 dz, \\ I &= \frac{3w-1}{2} \int_{-\infty}^{+\infty} \beta^2 dz - \frac{3w}{4} \int_{-\infty}^{+\infty} \beta^4 dz, \\ J &= \frac{1}{2} \int_{-\infty}^{+\infty} \beta^3 dz. \end{aligned} \tag{54}$$

The integrals are computed using usual methods. We get

$$\int_{-\infty}^{+\infty} \beta^2 dz = 8, \quad \int_{-\infty}^{+\infty} \beta^3 dz = 4\pi, \quad \int_{-\infty}^{+\infty} \beta^4 dz = \frac{64}{3}, \tag{55}$$

and hence

$$H = -2, \quad I = -4(w+1), \quad J = 2\pi. \tag{56}$$

Now we look for solutions of (53) of the form $\theta = \exp(i\omega Y + \lambda T)$, and

$$\lambda = \frac{-i\omega J \pm \sqrt{\Delta}}{2H}, \quad (57)$$

with

$$\Delta = \omega^2(4HI - J^2). \quad (58)$$

If Δ is positive, one of the solutions λ has a positive real part, and the solution is unstable. If $\Delta < 0$, λ is purely imaginary and no instability occurs. Therefore we see that the line-soliton is stable if its velocity w is less than

$$w_{th} = \frac{\pi^2}{8} - 1, \quad (59)$$

and unstable for $w > w_{th}$.

5. Numerical study

We intend to solve numerically the partial differential equations (28) and (29). The numerical scheme defined by

$$B(X_{j+1}, T_{j+1}) = B(X_j, T_{j+1}) + B(X_{j+1}, T_j) - B(X_j, T_j) + \delta X \delta T \partial_X \partial_T B(X_j + \delta X/2, T_j + \delta T/2), \quad (60)$$

with $X_j = X_0 + j\delta X$, $T_j = T_0 + j\delta T$ and so on, allows us to solve the system (28), (29) in the domain ($X \geq X_0$, $T \geq T_0$), for initial data given at ($T = T_0$) and boundary data given at ($X = X_0$). The problem is that the accuracy of the scheme (60) does not allow us to go far enough in the propagation of the soliton to observe the instability. Therefore we use another, more complicated, numerical scheme. We set $X_0 = 0$ and $T_0 = 0$ for simplicity. First equations (28) and (29) are integrated to yield

$$C_T(X, T) = \int_0^X [C_{YY}(X', T) + B_Y(X', T)] dX' - \frac{1}{2}B(X, T)^2 + C_T^b(T) + \frac{1}{2}B^b(T)^2, \quad (61)$$

$$B_T(X, T) = \int_0^X [C_X(X', T)B(X', T) + B_{YY}(X', T) - C_Y(X', T)] dX' + B_T^b(T), \quad (62)$$

where the boundary data

$$B^b(T) = B(X = 0, T), \quad (63)$$

$$C^b(T) = C(X = 0, T), \quad (64)$$

are given. The integro-differential system (61)–(62) together with the initial data

$$B^i(X) = B(X, T = 0), \quad (65)$$

$$C^i(X) = C(X, T = 0), \quad (66)$$

yield a Cauchy problem. It can be converted to a vectorial system of ordinary differential equations by semi-discretization in X , and this system can be solved using standard methods. Regarding the semi-discretization in X , we use the Simpson formula to compute the integrals and centred three-points finite differences formulae for the derivatives. Care must be taken

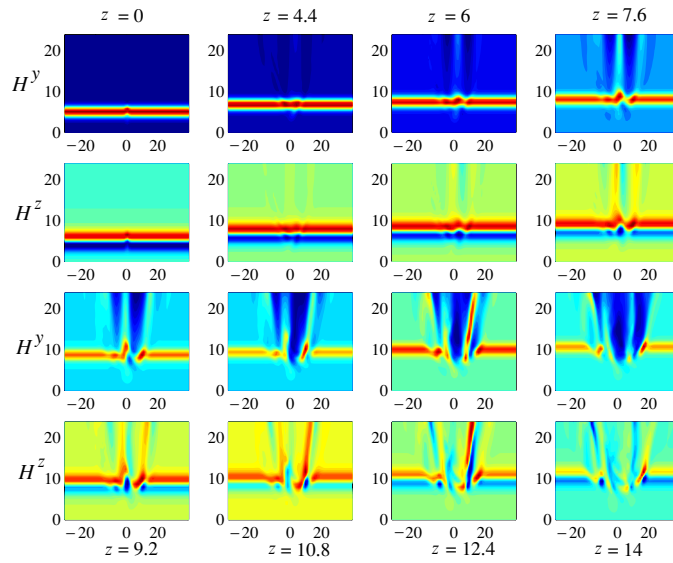


Figure 1. The evolution of the line-soliton in the unstable case. The relative soliton speed is $w = 0.4$.

at the ends of the box. The evolution in T is then solved using the classical fourth-order Runge–Kutta scheme.

The results of the numerical computations are as follows. The initial and boundary data correspond to a perturbed line-soliton are

$$B^i(X) = \frac{2w}{\cosh(X - X_1)}, \tag{67}$$

$$C^i(X) = w(2 \tanh(X - X_1) - X + X_1), \tag{68}$$

$$B^b(T) = \frac{2w}{\cosh(-wT - X_1)}, \tag{69}$$

$$C^b(T) = w(2 \tanh(-wT - X_1) + wT + X_1), \tag{70}$$

where

$$X_1 = X_0 + 0.2 \cos \frac{\pi y}{y_m} \tag{71}$$

describes the perturbation. y_m is the half length of the computation box in the y direction, X_0 is the position of the soliton at $T = 0$ and w is the soliton parameter equal to its relative velocity in the frame moving at speed $V = 1$, which is that of light in the medium.

The components of the magnetic field \vec{H} are computed from the numerical functions B and C using expression

$$\vec{H} \simeq \begin{pmatrix} 0 \\ H_0^y \\ H_0^z \end{pmatrix}, \tag{72}$$

with

$$H_0^y = -m(1 + C_X), \tag{73}$$

$$H_0^z = -mB_X, \tag{74}$$

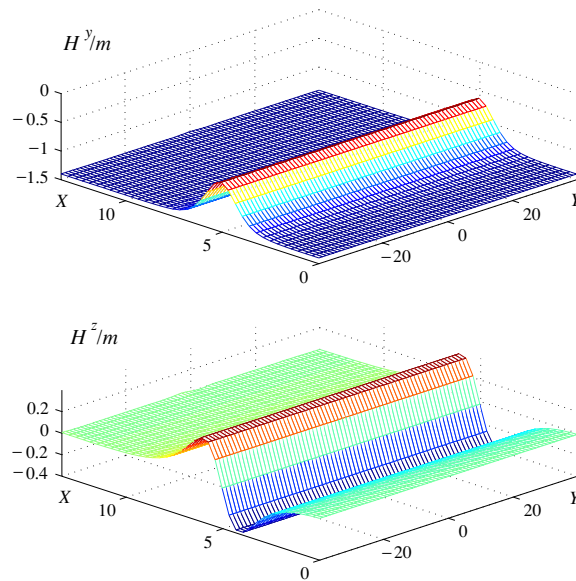


Figure 2. The stable line-soliton after propagation during $T = 30$. The relative soliton speed is $w = -0.4$.

and finite differences formulae. Expressions (74) follow straightforwardly from relations (16), (24) and (27) in section 3. The transverse instability found analytically in section 4 for $w > \pi^2/8 - 1$ is shown in figure 1 in the case $w = 0.4$. First the deformation of the line-soliton amplifies, then localized pulses begin to form. Afterwards, they go rapidly far away from the soliton, which loses almost all its energy through this process. The question whether stable structures can arise among these pulses, as it is the case for the Kadomtsev–Petviashvili (KP-I) equation [23], is left for further consideration. Stable propagation is predicted for $w < \pi^2/8 - 1$; it is numerically confirmed, as shown in figure 2, in the case $w = -0.4$.

6. Conclusion

We considered electromagnetic wave propagation in a saturated ferromagnetic dielectric. We have shown that under the influence of an external magnetic field, (2+1)-dimensional short electromagnetic waves can propagate. These waves are short along the propagation direction, which must be in a first approximation perpendicular to the external saturating magnetic field; they have a larger extension along the transversal direction.

An asymptotic model governing the nonlinear dynamics of the waves has been derived, it is a (2+1)-dimensional generalization of the completely integrable (1+1)-dimensional sine-Gordon equation. We give analytic solutions in the form of line-soliton excitations. The stability of the line-solitons with regard to transverse perturbations has been established both analytically and numerically. The stability criterion involves the soliton parameters, which measures both the relative speed of the soliton in the frame moving at the linear group velocity and its amplitude. The solitons are stable when they travel slower than some critical speed, which is close to the linear group velocity. Inclusion of damping in system (1), (2) and the study of the integrability properties of system (28), (29) remain the two more important open issues of our work. Indeed, the integrability of the (1+1)-dimensional case—the sine-Gordon

equation—does not allow to us conclude about the integrability of the (2+1)-dimensional system.

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